

Seeing Inside Storms

Blizzards can cause traffic accidents. Hurricanes can cause flooding. Tornadoes can destroy houses. Often, advance warning of these and other severe storms helps prevent deaths and reduce damage. For example, radio announcements can warn motorists to stay off roads, and municipal authorities can prepare to deal with possible flooding.

Giving advance warning is part of Dr. Paul Joe's work. Dr. Joe, a radar scientist and cloud physicist, is based at Environment Canada's radar site in King City, north of Toronto. Radar — short for *radio detection and ranging* — involves transmitting pulses of electromagnetic waves from an antenna. When objects such as snowflakes or raindrops interrupt these pulses, part of their electromagnetic energy is reflected back. A receiver picks up the reflections, converting them into a visible form and indicating a storm's location and intensity.



Dr. Paul Joe,
radar scientist and
cloud physicist

Conventional radar cannot detect a storm's internal motions, however. This is why, in recent years, Environment Canada has been improving its radar sites across the country by adding Doppler capability. This improved radar technology applies the Doppler effect: If an object is moving toward the radar, the frequency of its reflected energy is increased from the frequency of the

energy that the radar is transmitting. If an object is moving away from the radar, the frequency of its reflected energy is decreased.

"This is the same effect we notice with a subway train," Dr. Joe explains. "As it approaches, we hear a higher-pitched sound than when it leaves."

On Dr. Joe's radar screen, the frequency shifts are visualized using colours. In general, blue means an object is approaching; red means it is receding. But it's not that simple. Doppler images are complex and difficult for conventional weather forecasters to interpret, and Dr. Joe is working on ways to make them simpler. He also specializes in nowcasting — forecasting weather for the near future; for example, within an hour. As part of the 2000 Olympics, he went to Sydney, Australia, to join other scientists in demonstrating nowcasting technologies.

"I have it great," says Dr. Joe. "I love using what I've learned in mathematics, physics, and meteorology to decipher what Mother Nature is telling us and warning people about what she might do. Using the radar network, I can be everywhere chasing storms and seeing inside them in cyberspace."

Going Further

Dr. Joe's field, known in general as meteorology, includes radar science, cloud physics, climatology, and hydrometeorology. Research one of these fields and prepare a two-page report for presentation to the class.

WEB LINK

www.mcgrawhill.ca/links/physics12

The Canadian Hurricane Centre site maintained on the Internet by Environment Canada has a wide variety of information about hurricanes. Just go to the above Internet site and click on **Web Links**.

Electric Potential Difference

In previous physics courses, you learned that **electric potential difference** is the difference in the electric potential energy of a unit charge between two points in a circuit. You can broaden this definition to include any type of electric field, not just a field that is confined to an electric conductor. This concept allows you to describe the condition of a point in an electric field, relative to a reference point, without placing a charge at that point.

You have just derived an equation for the electric potential energy of a point charge, relative to infinity, a distance r from another point charge that can be considered as having created the field. For this case, you can find the electric potential difference between that point and infinity by considering the charge q_1 as the charge creating an electric field and q_2 as a unit charge.

- The definition of electric potential difference between a point and the reference point is $V = \frac{E_Q}{q_2}$
- Substitute the expression for the difference in electric potential energy of charge q_2 between the reference at infinity and the distance r from the charge q_1 due to the presence of q_1 . $V = \frac{\frac{kq_1q_2}{r}}{q_2}$
- Since only one q , the charge creating the field, remains in the expression, there is no need for a subscript. $V = \frac{kq}{r}$

ELECTRIC POTENTIAL DIFFERENCE DUE TO A POINT CHARGE

The electric potential difference, a scalar, between any point in the field surrounding a point charge and the reference point at infinity charge is the product of Coulomb's constant and the electric charge divided by the distance from the centre of the charge to the point.

$$V = k \frac{q}{r}$$

Quantity	Symbol	SI unit
electric potential difference	V	V (volts)
Coulomb's constant	k	$\frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$ (newton metres squared per coulombs squared)
electric charge	q	C (coulombs)
distance	r	m (metres)

Unit Analysis

$$\frac{\text{N} \cdot \text{m}^2 \cdot \cancel{\text{C}}}{\text{C}^2 \cdot \cancel{\text{m}}} = \frac{\text{N} \cdot \text{m}}{\text{C}} = \frac{\text{J}}{\text{C}} = \text{V}$$

PHYSICS FILE

Physicists often use the phrase, potential at a point, when they are referring to the potential difference between that point and the reference point an infinite distance away. It is not incorrect to use the phrase as long as you understand its meaning.

Problems involving electric potential difference can be extended, as can those involving electric field, to situations in which several source charges create an electric field. Since electric potential is a scalar quantity, the electric potential difference created by each individual charge is first calculated, being careful to use the correct sign, and then these scalar quantities are added algebraically.

You can go one step further and describe the electric potential difference between two points, P_1 and P_2 , within a field. To avoid confusion, this quantity is symbolized ΔV and the relationship is written as follows.

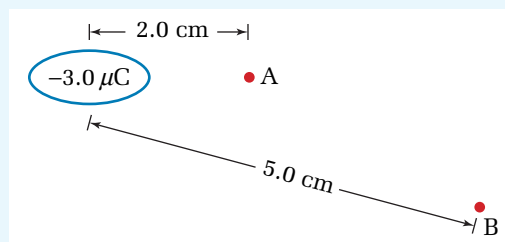
$$\Delta V = V_2 - V_1$$

Always keep in mind that V_1 and V_2 represent the electric potential difference between point 1 and infinity, and point 2 and infinity — a location so far away that the field is negligible. The following sample problems will help you to clarify these concepts in your mind.

SAMPLE PROBLEMS

Calculations Involving Electric Potential Difference

1. A small sphere with a charge of $-3.0 \mu\text{C}$ creates an electric field.
 - (a) Calculate the electric potential difference at point A, located 2.0 cm from the source charge, and at point B, located 5.0 cm from the same source charge.
 - (b) What is the potential difference between A and B?
 - (c) Which point is at the higher potential?



Conceptualize the Problem

- A *charged sphere* creates an *electric field*.
- At *any point* in the field, you can describe an *electric potential difference* between *that point* and a location an *infinite distance* away.
- Electric *potential difference* is a *scalar* quantity and depends only on the distance from the source charge and not the direction.
- The *potential difference* between *two points* is the *algebraic* difference between the individual potential differences of the points.

Identify the Goal

The electric potential difference, V , at each point

The electric potential difference, ΔV , between the two points

The point at a higher potential

Identify the Variables and Constants

Known

$$q = -3.0 \times 10^{-6} \text{ C}$$

$$d_A = 2.0 \times 10^{-2} \text{ m}$$

$$d_B = 5.0 \times 10^{-2} \text{ m}$$

Implied

$$k = 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

Unknown

$$V_A$$

$$V_B$$

Develop a Strategy

Use the equation for the electric potential difference at a point a distance r from a point charge.

Substitute numerical values and solve.

$$V_A = k \frac{q}{d_A}$$

$$V_A = \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}}{\text{C}^2} \right) \left(\frac{-3.0 \times 10^{-6} \text{ C}}{2.0 \times 10^{-2} \text{ m}} \right)$$

$$V_A = -1.35 \times 10^6 \text{ V}$$

$$V_A \cong -1.4 \times 10^6 \text{ V}$$

$$V_B = k \frac{q}{d_B}$$

$$V_B = \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}}{\text{C}^2} \right) \left(\frac{-3.0 \times 10^{-6} \text{ C}}{5.0 \times 10^{-2} \text{ m}} \right)$$

$$V_B = -5.4 \times 10^5 \text{ V}$$

- (a) The electric potential difference is $-1.4 \times 10^6 \text{ V}$ at point A, and $-5.4 \times 10^5 \text{ V}$ at point B.

Use algebraic subtraction to determine the potential difference between the two points.

$$\Delta V = V_B - V_A$$

$$\Delta V = (-5.4 \times 10^5 \text{ V}) - (-1.35 \times 10^6 \text{ V})$$

$$\Delta V = 8.1 \times 10^5 \text{ V}$$

- (b) The electric potential difference, ΔV , between points A and B is $8.1 \times 10^5 \text{ V}$.

Analyze the algebraic result and validate by considering the path of a positive test charge.

Algebraically, since $(V_B - V_A) > 0$, V_B is at the higher potential.

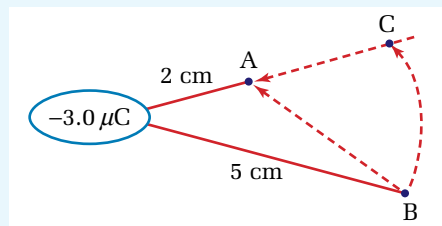
A positive test charge placed at point A would have to be dragged against the electric forces to get it to point B, which again places point B at the higher potential.

- (c) Point B is at the higher potential.

Validate the Solution

The more distant point has a smaller magnitude potential, but its negative sign makes it a higher value. The analysis with a positive test charge validates the statement of higher potential.

Note: The answers were obtained in this sample problem by taking into account whether the two points were on the same radial line. The diagram shows two possible paths a test charge could take in moving from B to A. If the test charge followed the path BCA, no work would be done on it from B to C, because the force would be perpendicular to the path.

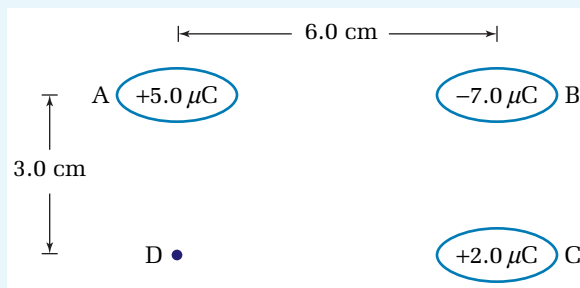


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The only segment of the path where work is done, and therefore the electric potential energy changed, is from C to A, parallel to the direction of the force acting.

- 2. The diagram shows three charges, A (+5.0 μC), B (−7.0 μC), and C (+2.0 μC), placed at three corners of a rectangle. Point D is the fourth corner. What is the electric potential difference at point D?**



Conceptualize the Problem

- There is an *electric potential difference* at point D, due to each of the separate charges.
- The separate potential values can be calculated and then *added algebraically*.

Identify the Goal

The electric potential difference, V , at point D

Identify the Variables and Constants

Known

$$\begin{aligned} q_A &= 5.0 \mu\text{C} \\ q_B &= -7.0 \mu\text{C} \\ q_C &= 2.0 \mu\text{C} \\ d_{AB} &= 6.0 \text{ cm} \\ d_{AD} &= 3.0 \text{ cm} \end{aligned}$$

Implied

$$\begin{aligned} k &= 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \\ d_{CD} &= 6.0 \text{ cm} \end{aligned}$$

Unknown

$$V_{\text{atD}}$$

Develop a Strategy

Calculate d_{BD} , using the Pythagorean theorem. Choose the positive value as a measure of the real distance.

$$d_{BD}^2 = (6.0 \text{ cm})^2 + (3.0 \text{ cm})^2$$

$$d_{BD}^2 = 45 \text{ cm}^2$$

$$d_{BD} = \pm\sqrt{45 \text{ cm}^2}$$

$$d_{BD} = \pm 6.7 \text{ cm}$$

Calculate the contribution of each charge to the potential difference at point D independently.

$$V_{A \text{ at D}} = \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \left(\frac{+5.0 \times 10^{-6} \text{ C}}{0.030 \text{ m}}\right) = 1.5 \times 10^6 \text{ V}$$

$$V_{B \text{ at D}} = \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \left(\frac{-7.0 \times 10^{-6} \text{ C}}{0.067 \text{ m}}\right) = -9.4 \times 10^5 \text{ V}$$

$$V_{C \text{ at D}} = \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \left(\frac{+2.0 \times 10^{-6} \text{ C}}{0.060 \text{ m}}\right) = 3.0 \times 10^5 \text{ V}$$

Calculate the net potential difference at point D by adding the separate potential differences algebraically.

$$V_{\text{atD}} = (1.5 \times 10^6 \text{ V}) + (-9.4 \times 10^5 \text{ V}) + (3.0 \times 10^5 \text{ V})$$

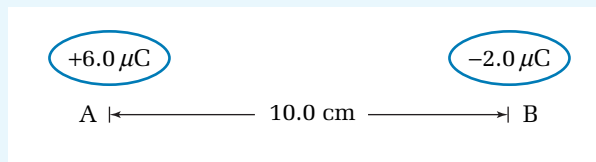
$$V_{\text{atD}} = 8.6 \times 10^5 \text{ V}$$

The electric potential difference at point D is $8.6 \times 10^5 \text{ V}$.

Validate the Solution

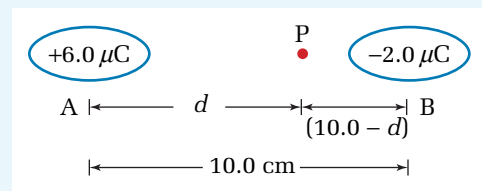
The electric potential difference contributed by A is expected to be stronger, due to its closer proximity and average charge.

3. A charge of $+6.0 \mu\text{C}$ at point A is separated 10.0 cm from a charge of $-2.0 \mu\text{C}$ at point B. At what locations on the line that passes through the two charges will the total electric potential be zero?



Conceptualize the Problem

- The *total electric potential* due to the combination of charges is the *algebraic sum* of the electric potential due to *each point* alone.
- Draw a diagram and assess the likely position.
- Let the points be designated a distance d to the right of point A, and set the absolute magnitudes of the potential equal to each other. This allows for two algebraic scenarios.



Identify the Goal

The location of the point of zero total electric potential

Identify the Variables and Constants

Known

$$q_A = +6.00 \times 10^{-6} \text{ C}$$

$$q_B = -2.00 \times 10^{-6} \text{ C}$$

$$d_{AB} = 10.0 \times 10^{-2} \text{ m}$$

Implied

$$k = 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

Unknown

d at zero total electric potential

Develop a Strategy

For the potentials to cancel algebraically, the point cannot be to the left of point A, which would be closer to the larger positive charge and could not be balanced by the potential of the negative charge. That leaves two locations: one between points A and B, and one to the right of point B, where the smaller distance to the negative charge balances the smaller value of that charge.

$$|V_{\text{due to A}}| = |V_{\text{due to B}}|$$

Scenario 1

$$k \frac{q_A}{d} = k \frac{q_B}{(0.10 - d)}$$

$$q_A(0.10 - d) = q_B(d)$$

$$0.10q_A - q_A d = q_B d$$

$$0.10q_A = d(q_A + q_B)$$

$$d = \frac{0.10q_A}{q_A + q_B}$$

$$d = \frac{(0.10 \text{ m})(6.0 \mu\text{C})}{6.0 \mu\text{C} + (-2.0 \mu\text{C})}$$

$$d = \frac{0.60 \text{ m} \cdot \mu\text{C}}{4.0 \mu\text{C}}$$

$$d = 0.15 \text{ m}$$

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Scenario 2

$$k \frac{q_A}{d} = -k \frac{q_B}{(0.10 - d)}$$

$$q_A(0.10 - d) = -q_B(d)$$

$$0.10q_A - q_Ad = -q_Bd$$

$$0.10q_A = d(q_A - q_B)$$

$$d = \frac{0.10q_A}{q_A - q_B}$$

$$d = \frac{(0.10 \text{ m})(6.0 \mu\text{C})}{6.0 \mu\text{C} - (-2.0 \mu\text{C})}$$

$$d = \frac{0.60 \text{ m} \cdot \mu\text{C}}{8.0 \mu\text{C}}$$

$$d = 0.075 \text{ m}$$

The points of zero potential are 7.5 cm to the right of point A and 5.0 cm to the right of point B. (Note: 15 cm to the right of A is the same as 5 cm to the right of B.)

Validate the Solution

The electric potentials due to point A at the two points are

$$(9.0 \times 10^9) \left(\frac{+6.0 \times 10^{-6}}{0.075} \right) = +7.2 \times 10^5 \text{ V} \text{ and } (9.0 \times 10^9) \left(\frac{+6.0 \times 10^{-6}}{0.15} \right) = +3.6 \times 10^5 \text{ V}$$

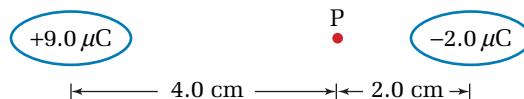
The electric potentials due to point B at the two points are

$$(9.0 \times 10^9) \left(\frac{-2.0 \times 10^{-6}}{0.025} \right) = -7.2 \times 10^5 \text{ V} \text{ and } (9.0 \times 10^9) \left(\frac{-2.0 \times 10^{-6}}{0.050} \right) = -3.6 \times 10^5 \text{ V}$$

In both locations, the potentials due to points A and B add algebraically to zero.

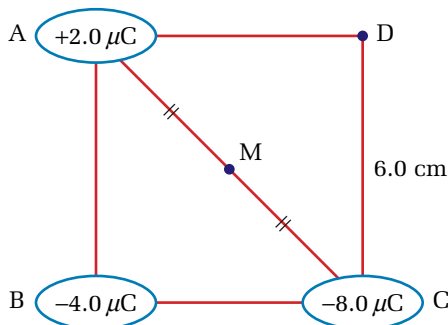
PRACTICE PROBLEMS

42. Find the electric field due to a point charge of $4.2 \times 10^{-7} \text{ C}$ at a point 2.8 cm from the charge.
43. How far from a positive point source of 8.2 C will the electric potential difference be 5.0 V? (**Note:** 8.2 C is a very large charge!)
44. The electric potential difference due to a point charge is 4.8 V at a distance of 4.2 cm from the charge. What will be the electric potential energy of the system if a second charge of $+6.0 \mu\text{C}$ is placed at that location?
45. The electric potential difference at a distance of 15 mm from a point charge is -2.8 V . What is the magnitude and sign of the charge?
46. Point charges of $+8.0 \mu\text{C}$ and $-5.0 \mu\text{C}$, respectively, are placed 10.0 cm apart in a vacuum. At what location along the line through them will the electric potential difference be zero?
47. What is the potential difference at point P situated between the charges $+9.0 \mu\text{C}$ and $-2.0 \mu\text{C}$, as shown in the diagram.

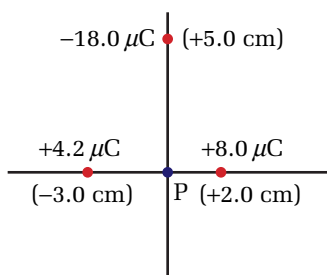


48. Point X has an electric potential difference of $+4.8 \text{ V}$ and point Y has a potential difference of -3.2 V . What is the electric potential difference, ΔV , between them?

49. Charges of $+2.0 \mu\text{C}$, $-4.0 \mu\text{C}$, and $-8.0 \mu\text{C}$ are placed at three vertices of a square, as shown in the diagram. Calculate the electric potential difference at M, the midpoint of the diagonal AC.



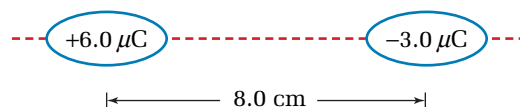
50. The diagram shows three small charges located on the axes of a Cartesian coordinate system. Calculate the potential difference at point P.



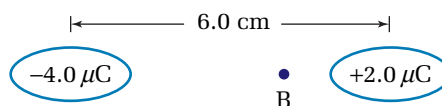
51. Two charges are placed at the corners of a square. One charge, $+4.0 \mu\text{C}$, is fixed to one corner and another, $-6.0 \mu\text{C}$, is fixed to the opposite corner. What charge would need to be placed at the intersection of the diagonals of the square in order to make the potential difference zero at each of the two unoccupied corners?
52. Point A has an electric potential difference of $+6.0 \text{ V}$. When a charge of 2.0 C is moved from point B to point A, 8.0 J of work are done on the charge. What was the electric potential difference of point B?
53. The potential difference between points X and Y is 12.0 V . If a charge of 1.0 C is released from the point of higher potential and allowed to move freely to the point of lower potential, how many joules of kinetic energy will it have?

54. Identical charges of $+2.0 \mu\text{C}$ are placed at the four vertices of a square of sides 10.0 cm . What is the potential difference between the point at the intersection of the diagonals and the midpoint of one of the sides of the square?

55. (a) If $6.2 \times 10^{-4} \text{ J}$ of work are required to move a charge of 3.2 nC (one nanocoulomb = 10^{-9} coulombs) from point B to point A in an electric field, what is the potential difference between A and B?
- (b) How much work would have been required to move a 6.4 nC charge instead?
- (c) Which point is at the higher electric potential? Explain.
56. Two different charges are placed 8.0 cm apart, as shown in the diagram. Calculate the location of the two positions along a line joining the two charges, where the electric potential is zero.



57. A charge of $+8.2 \text{ nC}$ is 10.0 cm to the left of a charge of -8.2 nC . Calculate the locations of three points, all of which are at zero electric potential.
58. A charge of $-6.0 \mu\text{C}$ is located at the origin of a set of Cartesian coordinates. A charge of $+8.0 \mu\text{C}$ is 8.0 cm above it. What are the coordinates of the points at which the potential is zero?
59. A charge of $+4.0 \mu\text{C}$ is 8.0 cm to the left of a point that has zero potential. Calculate three possible values for the magnitude and location of a second charge causing the potential to be zero.
60. Calculate the location of point B in the diagram below so that its electric potential is zero.



• Conceptual Problem

- In practice problem 56, do you think there could be locations (other than along a line joining the two charges) where the electric potential difference could be the same, but not zero? Explain.

Equipotential Surfaces

The quantities of gravitational potential energy, electric potential energy, and electric potential difference are all scalar quantities. Although it is rarely used, there is also a quantity called “gravitational potential difference,” which is defined as gravitational potential energy per unit mass. It is expressed mathematically as $V_g = \frac{E_g}{m} = -\frac{GM}{r}$. Since these are scalar quantities, the direction from the charge or mass that is creating the field does not affect the values. If you connected all of the points that are equidistant from a point mass or an isolated point charge, they would have the same potential difference and they would be creating a spherical surface. Such a surface, illustrated in Figure 7.17, is called an **equipotential surface**.

equipotential surfaces

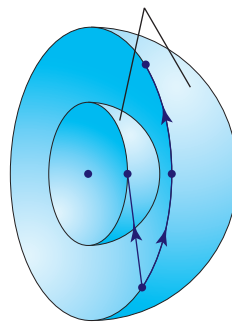


Figure 7.17 The spherical shells could represent equipotential surfaces either for a gravitational field around a point mass (or spherical mass) or for an electric field around an isolated point charge. In cross section, the equipotential spherical surfaces appear as concentric circles.

GEOGRAPHY LINK

The equipotential lines around a system of charges could be compared to the contour lines on topographical maps. Since these contour lines represent identical heights above sea level, they also represent points that have the same gravitational energy per unit mass, and so are equipotential lines.

$$E_g = mgh$$

$$\frac{E_g}{m} = gh$$

$$V_g \propto h$$

You will recall that the work done per unit charge in moving that charge from a potential V_1 to a potential V_2 is $\frac{W}{q} = V_2 - V_1$.

Since, on an equipotential surface, $V_1 = V_2$, the work done must be zero. In other words, no work is required to move a charge or mass around on an equipotential surface, and the electric or gravitational force does no work on the charge or mass. Consequently, a field line must have no component along the equipotential surface. An equipotential surface must be perpendicular to the direction of the field lines at all points. Figure 7.18 shows the electric field lines and equipotential surfaces for pairs of point charges.

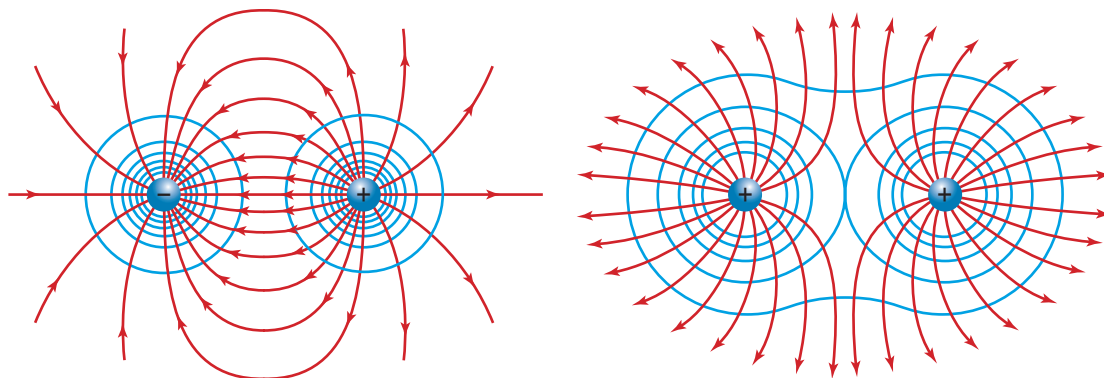


Figure 7.18 The field lines for these electric dipoles are shown in red and the cross section of the equipotential surfaces are in blue. Notice that field lines are always perpendicular to equipotential surfaces.

• Conceptual Problem

- Could the barometric lines on a weather map be considered to be equipotential lines?

7.3 Section Review

1. **K/U** What are the differences in the data required to calculate the gravitational potential energy of a system and the electric potential energy?
2. **K/U** How does the amount of work done relate to the electric potential difference between two points in an electric field?
3. **MC** Research and briefly report on the use of electric potential differences in medical diagnostic techniques such as electrocardiograms.
4. **MC** Research and report on the role played by electric potential differences in the transmission of signals in the human nervous system.
5. **C** Can an equipotential surface in the vicinity of two like charges have a potential of zero? Explain the reason for your answer.
6. **I** Investigate Internet sites that use computer programs to draw the electric field lines near a variety of charge systems. Prepare a portfolio of various patterns.
7. **K/U** How could you draw in the equipotential surfaces associated with the patterns obtained in question 6?
8. **I**
 - (a) Why do you think atomic physicists tend to speak of the electrons in atoms as having “binding energy”?
 - (b) Investigate the use of the term “potential well” to describe the energy state of atoms.